Voting in Assemblies of Shareholders and Incomplete Markets[*]

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Abstract

An economy with two dates is considered, one state at the first date and a finite number of states at the last date. Shareholders determine production plans by voting – one share, one vote – and at ρ-majority stable equilibria, alternative production plans are supported by at most $\rho \times 100$ percent of the shareholders. It is shown that a ρ-majority stable equilibrium exists provided that

$$\rho \geq \min\left\{\frac{S-J}{S-J+1}, \frac{B}{B+1}\right\}$$

where S is the number of states at the last date, J is the number of firms and B is the dimensions of the sets of efficient production plans for firms. Moreover, an example shows that ρ-majority stable equilibria need not exist for smaller ρ's.

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1 Introduction

If markets are complete then consumers have common shadow prices { namely the vector of market prices. So shareholders agree that ¯rms should maximize pro¯ts with respect to these common prices. However, if markets are incomplete, shadow prices need not be common. Thus, typically shareholders disagree on the production plans to be chosen. Therefore several suggestions have been put forward as reasonable objectives for ¯rms.

It seems natural that production plans should satisfy the Pareto criterion: there are no alternative production plans that make some shareholders better o® and none worse o®. Unfortunately, the Pareto criterion is weak: production plans satisfy the Pareto criterion if and only if they maximize pro¯ts with respect to some price vector in the convex hull of the shareholders' shadow prices.

Drµeze (1974) and Grossman & Hart (1979) agree that production plans should satisfy the Pareto criterion and propose that sidepayments between shareholders be allowed. Drµeze (1974) (resp. Grossman & Hart (1979)) suggests that production plans should re°ect preferences of ¯nal (resp. initial) shareholders: this may be interpreted as production plans are determined after markets close (resp. before markets open). However, sidepayments depend on information that shareholders have incentives to manipulate, a weakness that voting rules overcome.

Drµeze (1985) suggests that production plans should be stable for simple majority voting between shareholders and unanimity between board members (without sidepayments): there is no alternative production plan that makes all board members as well as a majority of shareholders better o®. As in Drµeze (1974) production plans re°ect preferences of ¯nal shareholders. It appears to be a drawback that unanimity between board members is essential for existence of equilibria.

DeMarzo (1993) investigates some properties of equilibria where production plans are stable for simple majority voting between shareholders. Typically the largest shareholder determines the production plan at these equilibria. Also, DeMarzo shows that stability for simple majority voting between shareholders and unanimity between board members imply that board members determine the production plan. However, as he argues, such equilibria need not exist unless either the degree of market incompleteness or the dimension of the set of efficient production plans is 1.

In the present paper, stability with respect to ρ-majority voting between shareholders is studied: there is no alternative production plan that makes more than $\rho \times 100$ percent of the shareholders better off. Indeed, at a ρ-majority stable equilibrium (or ρ-MSE), consumers do not want to change their portfolios, firms are not able to make more than $\rho \times 100$ percent of their shareholders better off by changing production plans and finally, markets clear. It is shown that if portfolios are unbounded then a ρ-MSE exists provided that

$$\rho \geq \min \left\{ \frac{S-J}{S-J+1}, \frac{B}{B+1} \right\}$$

where S is the number of states at the last date, J is the number of firms and B is the dimension of the set of efficient production plans for firms. If portfolios are bounded to be non-negative then a ρ-MSE exists provided that $\rho \geq B/(B+1)$.

Different timings of trade and vote are considered. Voting may take place while markets are open or after markets close, in which case final shareholders vote (as in Drèze (1985) and DeMarzo (1993)). And it may take place before markets open, in which case initial shareholders vote. In case of voting before markets open or while they are open, shareholders need to form expectations about price variations. Two types of price perceptions are considered: competitive price perceptions (as introduced by Grossman & Hart (1979)) and fixed price perceptions. According to competitive price perceptions consumers perceive that income vectors are valued by their shadow prices; whereas according to fixed price perceptions they perceive that prices are not influenced by changes in production plans.

In general, changes of production plans in°uence trading opportunities through two channels: they change the value of portfolios as well as the span of assets. From this perspective, competitive price perceptions and ¯xed price perceptions represent two extremes: consumers concentrate on how changes of production plans change the value of their portfolios with the former and the span of assets with the latter.

In case markets are complete, a ½-MSE exists even for unanimity, i.e. with ½ = 0. Ekern & Wilson (1974) have shown that this result extends to the case of partial spanning, i.e. the sets of e±cient production plans are subsets of the span of assets[1]. In case markets are incomplete such that either the degree of incompleteness is 1 or the sets of e±cient production plans are 1-dimensional, a ½-MSE exists for simple majority voting, i.e. with ½ = 1=2, as argued by DeMarzo (1993). It is shown here that in case of a more severe degree of incompleteness and higher dimensions of the sets of e±cient production plans, super majority rules (½ > 1=2) are needed to ensure existence of ½-MSE.

The social choice literature o®ers some general results on existence of stable equilibria under super majority voting { see, e.g., Ferejohn & Grether (1974), Greenberg (1979), Caplin & Nalebu® (1988, 1991) and Balasko & Crµs (1997). Crµs (2000) exploits the results of Caplin & Nalebu® (1988, 1991) to obtain some conditions on the distribution of consumers' characteristics under which a ½-MSE exists for ½ between 0.5 and 0.64 in a model with a continuum of consumers, restrictive assumptions on production sets and preferences of consumers. Here the result of Greenberg (1979) is exploited to obtain a lower bound on the rate ½ for which a ½-MSE exists, in a model with a ¯nite number of consumers, weak assumptions on productions sets and preferences, and no assumptions on the distribution of consumers' characteristics. A di±culty in applying the results from the social choice

[1]See Magill & Quinzii (1996), chapter 6. Actually, existence of ½-MSE for ½ = 0 holds in any model with incomplete markets where equilibrium allocations are Pareto optimal, e.g., under strong conditions for the CAPM (Borch (1968) and Wilson (1968)).

literature is that preferences of shareholders, as well as shares (i.e. voting weights), are endogeneously determined through general equilibrium effects.

Even though the proposed bounds on ρ are quite high and cannot be improved, as shown by an example, the results of the present paper show that: (1) the degree of market incompleteness plays a fundamental role in restricting the dimension of the set of alternatives and thereby in aggregating preferences of shareholders and (2) the lower the degree of market incompleteness the lower super majority rate is necessary to ensure existence of a ρ-MSE.

The paper is organized as follows: in Section 2 the model is introduced; in Section 3 assumptions are stated, existence of a ρ-MSE, for $\rho \geq \min\{(S-J)/(S-J+1); B/(B+1)\}$, is established in case voting takes place after markets close, and an example is given showing that the latter bound cannot be improved; in Section 4 price perceptions are introduced and existence of a ρ-MSE is established in case voting takes place either before markets open or while they are open, and; finally Section 5 contains some concluding remarks. All proofs are gathered in an appendix.

2 The model

Consider an economy with 2 dates, $t \in \{0; 1\}$, 1 state at the first date, $s = 0$, and S states at the second date, $s \in \{1; \ldots; S\}$. There are: 1 commodity at every state, I consumers, $i \in \{1; \ldots; I\}$, and J firms, $j \in \{1; \ldots; J\}$. Consumers are characterized by their consumption sets, $X_i \subset \mathbb{R}^{S+1}$, endowments, $\omega_i \in \mathbb{R}^{S+1}$, preferences described by correspondences, $P_i : X_i \to X_i$, and initial portfolio of shares in firms, $\delta_i \in \mathbb{R}^J$ where $\sum_{i=1}^{I} \delta_{ij} = 1$ for all j. Firms are characterized by their production sets, $Y_j \subset \mathbb{R}^{S+1}$.

Consumers choose consumption plans, $x_i \in X_i$, and portfolios, $\theta_i \in \mathbb{R}^J$. Firms choose production plans, $y_j \in Y_j$. For convenience, let $x = (x_i)_{i=1}^{I}$, $\theta = (\theta_i)_{i=1}^{I}$, $y = (y_j)_{j=1}^{J}$, $q = (q_j)_{j=1}^{J} \in \mathbb{R}^J$ where q_j is the price of shares in firm j, $Y = (y_1 \cdots y_J)$ and

$$Q = \begin{pmatrix} q_1 & \cdots & q_J \\ 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{pmatrix}.$$

With some abuse of notation, q_j denotes the price of shares in firm j as well as the j'th column of Q. The budget set of consumer i is

$$B_i(Q;Y) = \{x_i \in X_i | x_i \leq \omega_i + Q\delta_i + (Y - Q)\theta_i \text{ for some } \theta_i \in R^J\}$$

and x_i is a solution to the problem of consumer i provided that $x_i \in B_i(Q;Y)$ and $P_i(x_i) \cap B_i(Q;Y) = \emptyset$. Hence, there are no strategic considerations involved in the choices of portfolios. Let $U_{ij}(x_i;\theta_{ij};y_j)$ denote the set of production plans for firm j that make, at the considered allocation $(x;\theta;y)$, consumer i better off, i.e.

$$U_{ij}(x_i;\theta_{ij};y_j) = \{y'_j \in Y_j | x_i + (y'_j - y_j)\theta_{ij} \in P_i(x_i)\}.$$

Next let $u_j(x;\theta_j;y_j;y'_j)$ denote the set of consumers who are, at the considered allocation $(x;\theta;y)$, better off with production plan y'_j for firm j rather than y_j, i.e.

$$u_j(x;\theta_j;y_j;y'_j) = \{i \in \{1;\dots;I\} | y'_j \in U_{ij}(x_i;\theta_{ij};y_j)\}.$$

Then preferences of firms are described, for a fixed rate ρ of the super majority rule, by correspondences, $P_j^\rho : \prod_i X_i \times R^l \times Y_j \to Y_j$, defined by

$$P_j^\rho(x;\theta_j;y_j) = \begin{cases} \emptyset & \text{for } \sum_i \theta_{ij}^+ = 0 \\ \left\{y'_j \in Y_j \,\middle|\, \dfrac{\sum_{i \in u_j(x;\theta_j;y_j;y'_j)} \theta_{ij}^+}{\sum_i \theta_{ij}^+} > \rho\right\} & \text{for } \sum_i \theta_{ij}^+ > 0. \end{cases}$$

And y_j is a solution to the problem of firm j provided that $P_j^\rho(x;\theta_j;y_j) \cap Y_j = \emptyset$. Thus, if the production plan is changed from y_j to y'_j then this change is distributed to shareholders propotionally to their shares.

Definition 1 $(q^*, x^*, \theta^*, y^*)$ is a **ρ-majority stable equilibrium** provided that

- $x_i^* \leq \omega_i + Q^*\delta_i + (Y^* - Q^*)\theta_i^*$ and x_i^* is a solution to the problem of consumer i for (q^*, y^*), i.e.

$$x_i^* \in B_i(Q^*, Y^*) \text{ and } P_i(x_i^*) \cap B_i(Q^*, Y^*) = \emptyset$$

  for all $i \in I$,

- y_j^* is a solution to the problem of firm j for (x^*, θ_j^*), i.e.

$$y_j^* \in Y_j^* \text{ and } P_j^\rho(x^*, \theta_j^*, y_j^*) \cap Y_j^* = \emptyset$$

  for all $j \in J$, and,

- markets clear, i.e.

$$\sum_i x_i^* = \sum_i \omega_i^* + \sum_j y_j^* \text{ and } \sum_i \theta_{ij}^* = 1$$

  for all $j \in J$.

3 Assumptions and existence of equilibrium

Assumptions on consumers, firms and the production sector are imposed in order to ensure the existence of a ρ-majority stable equilibrium.

Consumers are supposed to satisfy the following assumptions

(a.1) $X_i = \mathbb{R}^{S+1}$,

(a.2) $\omega_i \in \mathbb{R}^{S+1}$,

(a.3) $\mathrm{gr}\, P_i$ is open,

(a.4) $\{x_i\} + \mathbb{R}_+^{S+1} \setminus \{0\} \in P_i(x_i)$, (a.5) for all x_i, there exists a unique $\pi_i \in \Delta_+$ such that $\pi_i \cdot (x_i' - x_i) > 0$ for all $x_i' \in P_i(x_i)$ where $\Delta_+^S = \{\lambda \in \mathbb{R}_+^{S+1} | \sum_s \lambda^s = 1\}$, and,

(a.6) if $A \subset \mathbb{R}^{S+1}$ is compact then there exists $x_i(A) \in \mathbb{R}^{S+1}$ such that if $x_i \in \mathbb{R}^{S+1} \setminus (\{x_i(A)\} + \mathbb{R}_+^{S+1})$ then $A \subset P_i(x_i)$.

Assumptions (a.1) and (a.2) imply that consumption sets are unbounded as considered by Balasko (1988) while assumptions (a.3), (a.4), (a.5) and (a.6) are generalizations of equivalent assumptions considered by Balasko (1988) to non-transitive, non-complete and non-differentiable preferences. Assumptions (a.3) and (a.4) are standard continuity and monotonicity assumptions; assumption (a.5) states existence of a unique shadow price vector $\pi_i(x_i)$ at each consumption bundle x_i, and; assumption (a.6) generalizes the standard "boundedness from below" property of indifference sets to the present framework where preferences are not necessarily transitive nor complete.

Let $Z_j \subset \mathbb{R}^{S+1}$ be the set of efficient production plans, i.e.

$$Z_j = \{y_j \in \mathbb{R}^{S+1} | (\{y_j\} + \mathbb{R}_+^{S+1}) \cap Y_j = \{y_j\}\}$$

then firms are supposed to satisfy the following assumptions

(a.7) the production set, Y_j, is convex and closed, and,

(a.8) there exists a compact and B-dimensional affine set, $B_j \subset \mathbb{R}^{S+1}$, such that $Z_j \subset B_j$.

Assumption (a.7) is standard while assumption (a.8) includes "truncated" production sets such as

$$\{y \in \mathbb{R}^{S+1} | y^0 \in [\bar{y}, 0] \text{ and } y^s \le (y^0)^b \text{ for all } s \in \{1, \ldots, S\}\}$$

where $\bar{y} \le 0$ and $b \in]0, 1]$.

Moreover, the production sector of the economy is supposed to satisfy the following assumption (a.9) production plans for date 1, $((y_j^s)_{s=1}^S)_{j=1}^J$, are linearly independent for all production plans in the convex hull of the closure of the set of e±cient production plans, y_j 2 co cl Z_j for all j.

Assumption (a.9) excludes that ¯rms are able to replicate production plans of each other.

Theorem 1 There exists a ½-majority stable equilibrium for all economies which satisfy assumptions (a.1) to (a.9) if and and only if

$$½ \;¸\; \min \text{½} \frac{S ¡ J}{S ¡ J + 1}; \frac{B}{B + 1} \text{¾} :$$

Remark: The argument to establish the \if" of the assertion is based on the proofs of Theorem 2 in Greenberg (1979) and the theorem in Shafer & Sonnenschein (1975). A generalized game is constructed where, among other constructions, ¯rms determine production plans that maximize pro¯ts with respect to prices which re°ect interests of their shareholders and groups of shareholders (one per ¯rm) determine prices for which ¯rms maximize pro¯ts. Hence, the original problem of the ¯rm { which is to ¯nd a production plan for which no alternative production plan can be supported by a ½-majority of its shareholders { is decomposed into pro¯t maximization with respect to ¯rm speci¯c prices and determination of ¯rm speci¯c prices with respect to some arti¯cial preferences for its shareholders.

The argument to establish the \only if" of the assertion is based on the construction of an economy for which no ½-majority stable equilibrium with ½ < minf(S ¡ J)=(S ¡ J + 1); B=(B + 1)g exists.

End of remark

In case S ¡ J = 1, Theorem 1 ensures existence of a simple majority stable equilibrium. It is easily seen that the prices for which ¯rms maximize pro¯ts are, in this case, typically not the ones Drµeze (1974) suggests. Indeed, in theorem 1 the shadow price vector of the median shareholder is used whereas Drµeze (1974) suggests that the average shadow price vector should be used.

Trading on the financial markets, when consumers are not constrained in their portfolio choices, leads to suitable normalized shadow prices being contained in some $(S - J)$-dimensional affine set $(\langle Y^* - Q^* \rangle^{\perp} \cap \Delta^S_+)$. However, if there are restrictions on portfolios, like short sales constraints, then the degree of market incompleteness need not restrict shadow prices.

Corollary 1 Suppose that portfolios are bounded such that $\theta_i \in [0;1]^J$ for all i and that $\operatorname{co} \operatorname{cl} Z_j \subset \mathbb{R}^{S+1}_+$ for all j. Then there exists a ρ-majority stable equilibrium provided that

$$\rho \geq \frac{B}{B+1}.$$

It is hard to love the assumption that $\operatorname{co} \operatorname{cl} Z_j \subset \mathbb{R}^{S+1}_+$ for all j. However, the "Cass-trick" – one consumer trades on complete markets – cannot be applied in corollary 1 because portfolios are bounded to be between 0 and 1. Therefore existence of equilibrium is only ensured provided that prices of shares are positive as explained by Radner (1972) and the assumption ensures this.

4 Price perceptions

In the present section different timings between trade and vote are considered. At a ρ-majority stable equilibrium, $(q^*; x^*; \theta^*; y^*)$, if consumer i considers how to vote with regard to a change from $(q^*_j; y^*_j)$ to $(q_j; y_j)$ of price and production plan for firm j (where $\delta_{ij} > 0$ or $\theta^*_{ij} > 0$ because otherwise consumer i have no voting weight) then

- in case voting takes place after markets close, she votes for the change if and only if

$$x^*_i + (Y^*|y_j - Y^*)\theta^*_i \in P_i(x^*_i)$$

where $Y^*|y_j$ is Y^* with y_j replacing y^*_j,

- in case voting takes place while markets are open, she votes for the change if and only if

$$x_i^* - (Y^* - Q^*|q_j)\theta_i^* + (Y^*|y_j - Q^*|q_j)\theta_i \in P_i(x_i^*)$$

  for some θ_i, and,

- in case voting takes place before markets open, she votes for the change if and only if

$$x_i^* - (Q^* - Q^*|q_j)\delta_i - (Y^* - Q^*)\theta_i^* + (Y^*|y_j - Q^*|q_j)\theta_i \in P_i(x_i^*)$$

  for some θ_i (here the voting weights are δ_j^+).

If portfolios are unbounded, i.e. $\theta_i \in R^J$, then $\pi_i(x_i^*) \in \langle Y^* - Q^*\rangle^\perp$ at a ρ-majority stable equilibrium, $(q^*; x^*; \theta^*; y^*)$. Therefore in case voting takes place after markets close (resp. while markets are open or before they open), if consumer i votes for the change then $\pi_i(x_i^*) \cdot (y_j - y_j^*) > 0$ (resp. $\pi_i(x_i^*) \cdot (q_j - q_j^*) > 0$ or $\pi_i(x_i^*) \cdot (y_j - q_j) \neq 0$). Thus, equivalently, in case voting takes place after markets close (resp. while markets are open or before they open), if $\pi_i(x_i^*) \cdot (y_j - y_j^*) \leq 0$ (resp. $\pi_i(x_i^*) \cdot (Q - Q^*) \leq 0$ and $\pi_i(x_i^*) \cdot (y_j - q_j) = 0$) then consumer i votes against the change. However, consumers do not know how prices depend on production plans so if voting takes place before or while markets are open then they need to form perceptions about this.

4.1 Competitive price perceptions

Grossman & Hart (1979) introduced the notion of competitive price perceptions in a model where production plans are determined by shareholders before markets open. Consider a ρ-majority stable equilibrium, $(q^*; x^*; \theta^*; y^*)$, then a change of production plan from y_j^* to y_j for firm j is perceived by consumer i to change the price from q_j^* to

$$q_{ij}(x_i^*; y_j) = \frac{1}{\pi_i^0(x_i^*)} \pi_i(x_i^*) \cdot y_j:$$

Consequently, if consumer i votes for the change and has competitive price perception then $\mu_i(x_i^*) \cdot (y_j - y_j^*) > 0$ and, equivalently, if $\mu_i(x_i^*) \cdot (y_j - y_j^*) \leq 0$ then consumer i votes against the change. This does not depend on whether voting takes place before markets open, while they are open or after they close. Informally, if consumers have competitive price perceptions then they concentrate on how changes of production plans change values of their portfolios rather than the span of assets.

Thus, there is a different interpretation of the model of Section 2 as well as the results of Section 3: voting takes place while markets are open and consumers have competitive price perceptions. Moreover, Theorem 1 and Corollary 1 extend to the model with voting before markets open provided that consumers have competitive price perceptions. However in this latter case the set of equilibria is typically not identical to the set of equilibria of the former case because voting weights typically are not identical, i.e. $\theta_i^* \neq \delta_i$. Hence, the next corollary follows from the proof of Theorem 1 with only minor modifications.

Corollary 2 Suppose that voting takes place before markets open, while they are open or after they close and that consumers have competitive price perceptions. Then there exists a ρ-majority stable equilibrium provided that

$$\rho \geq \min\left\{\frac{S - J}{S - J + 1}, \frac{B}{B + 1}\right\}.$$

4.2 Fixed price perceptions

Consider a ρ-majority stable equilibrium, $(q^*; x^*; \theta^*; y^*)$, then a change of production plan from y_j^* to y_j for firm j is perceived by consumer i not to change the price, q_j^*. Informally, if consumers have fixed price perceptions then they concentrate on how changes of production plans change the span of assets rather than values of portfolios.

Portfolios are bounded between 0 and 1, i.e. $\theta_i \in [0; 1]^J$, so the modified budget set of consumer i is

$$\breve{B}_i(Q;Y) = \{x_i \in X_i | x_i \leq \omega_i + Q\delta_i + (Y - Q)\theta_i \text{ for some } \theta_i \in [0;1]^J\}.$$

Let

$$\begin{aligned}\breve{U}_{ij}(q;x_i;y) &= \{y'_j \in Y_j | x_i + (Y | y'_j - Q)\theta_i \in P_i(x_i) \\ &\qquad \text{for some } \theta_i \in [0;1]^J\} \\ \mathcal{U}_j(q;x;y;y'_j) &= \{i \in \{1;\dots;I\} | y'_j \in \breve{U}_{ij}(q;x_i;y_j)\}\end{aligned}$$

then preferences of firms are described by correspondences, $\breve{P}_j^{\rho} : R^J \times \prod_i X_i \times [0;1]^I \times \prod_j Y_j \to Y_j$, defined by

$$\breve{P}_j^{\rho}(q;x;\theta_j;y) = \begin{cases} \emptyset; & \text{for } \sum_i \theta^+_{ij} = 0 \\ \{y'_j \in Y_j | \frac{\sum_{i \in \mathcal{U}_j(x;\theta_j;y;y'_j)} \theta^+_{ij}}{\sum_i \theta^+_{ij}} > \rho\} & \text{for } \sum_i \theta^+_{ij} > 0 \end{cases}$$

in case voting takes place while markets are open and δ_j replaces θ_j in case voting takes place before markets open.

Corollary 3 Suppose that portfolios are bounded such that $\theta_i \in [0;1]^J$ for all i, that co cl $Z_j \subset R^{S+1}_+$ for all j and that consumers have fixed price perceptions. Then a ρ-majority stable equilibrium exists provided that

$$\rho \geq \frac{B}{B+1}.$$

5 Final remarks

In the present paper, bounds on ρ are provided such that ρ-majority stable equilibria exist. To complement these results on existence of equilibrium it would be nice study: (1) the efficiency properties of equilibrium allocations, and; (2) the "size" of the set of equilibria.

On the one hand, in many countries, simple majority voting is used in assemblies of shareholders. On the other hand, the provided bounds on ½ implies that simple majority stable equilibria need not exist unless either the degree of incompleteness is 1 or the sets of e±cient production plans are 1-dimensional. Therefore it would be interesting to ¯nd \reasonable" assumptions on production sets and preferences of consumers that ensure existence of ½-majority stable equilibria for lower values of ½.

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Appendix

Proof of Theorem 1

In part 1, resp. part 2, it is shown that if $\rho \geq (S - J)/(S - J + 1)$, resp. $\rho \geq B/(B + 1)$, then a ρ-majority stable equilibrium exists. In part 3, an example is provided of an economy for which no ρ-majority stable equilibrium with $\rho < \min\{(S - J)/(S - J + 1); B/(B + 1)\}$ exists.

Part 1: $\rho \geq (S-J)/(S-J+1)$

The variables to be determined are state prices, $\lambda \in \Delta^S_+$, consumption bundles for consumers, $x = (x_i)_{i=1}^I \subset \prod_{i=1}^I X_i$, production plans for firms, $y = (y_j)_{j=1}^J \subset \prod_{j=1}^J Y_j$, and prices with respect to which firms maximize profits, $\pi = (\pi_j)_{j=1}^J \in \prod_{j=1}^J \Delta^S_+$.

The auctioneer (agent 0) determines state prices in order to maximize the value of excess demand. Consumers (agent $k \in \{1, \dots, I\}$) determine maximal consumption bundles for their preferences. Firms (agent $k \in \{I+1, \dots, I+J\}$) determine production plans that maximize profits with respect to prices which reflect interests of their shareholders. Groups of shareholders (agent $k \in \{I+J+1, \dots, I+2J\}$, one group per firm) determine prices for which firms maximize profits. Hence, the original problem of the firm – which is to find a production plan for which no alternative production can be supported by a ρ-majority of its shareholders – is decomposed into profit maximization with respect to firm specific prices and determination of firm specific prices with respect to some artificial preferences for its shareholders.

In a first step, these four categories of agents (auctioneer, consumers, firms and groups of shareholders) are described. In a second step, a suitable correspondence is constructed and Kakutani's fixed point theorem is applied. Finally, in a third step, the fixed point is shown to be a ρ-majority stable equilibrium.

Step 1: description of agents

"Auctioneer" For agent $k = 0$, the strategy set, $V_k \subset \mathbb{R}^{S+1}$, is defined by

$$V_k = \Delta^S_+ \cap [\frac{1}{n}; 1]^{S+1}$$

where $n \in \mathbb{N}$, the constraint correspondence, $C_k : V \to V_k$ (where V is the product of the agents' strategy sets, to be defined in the sequel), is defined by

$$C_k(\lambda; x; y; \pi) = V_k$$

and the preference correspondence, $Q_k : V \to V_k$, is defined by

$$Q_k(\lambda;x;y;\pi) \;=\; \{\lambda' \in V_k \,|\, (\lambda' - \lambda)\cdot(\sum_i x_i - \sum_i \omega_i - \sum_j y_j) > 0\}.$$

Clearly, V_k is compact and convex, C_k is continuous and gr Q_k is open with $\lambda \notin \mathrm{co}\, Q_k(\lambda;x;y;\pi)$.

"Consumers" For agent $k \in \{1;\ldots;I\}$, the strategy set, $V_k \subset X_i$ where $i = k$, is defined by

$$V_k \;=\; X_i \cap (\{\omega_i\} + \sum_j \delta_{ij} \mathrm{co}\,\mathrm{cl}\, Z_j + [-n;n]^{S+1})$$

where $n \in N$, the constraint correspondence, $C_k : V \to V_k$, is defined by

$$C_k(\lambda;x;y;\pi) \;=\; \{x'_i \in V_k | \lambda\cdot(x'_i - \omega_i - Q\delta_i) \le 0\}$$

for $k = 1$ where $q_j = (1/\lambda^0)\lambda\cdot y_j$ for all j and

$$C_k(\lambda;x;y;\pi) \;=\; B_i(Y;Q) \cap V_k$$

for $k \in \{2;\ldots;I\}$ and the preference correspondence, $Q_k : V \to V_k$, is defined by

$$Q_k(\lambda;x;y;\pi) \;=\; \mathrm{co}\, P_i(x_i) \cap V_k.$$

Clearly, V_k is compact and convex, C_k is continuous and gr Q_k is open with $x_i \notin \mathrm{co}\, Q_k(\lambda;x;y;\pi)$ for $i = k$.

"Firms" For agent $k \in \{I+1;\ldots;I+J\}$, the strategy set, $V_k \subset Y_j$, is defined by

$$V_k \;=\; \mathrm{co}\,\mathrm{cl}\, Z_j$$

where $j = k - I$, the constraint correspondence, $C_k : V \to V_k$, defined by

$$C_k(\lambda;x;y;\pi) \;=\; V_k$$

and the preference correspondence, $Q_k : V \to V_k$, defined by

$$Q_k(\lambda;x;y;\pi) \;=\; \{y'_j \in Z_j | \pi_j\cdot(y'_j - y_j) > 0\} \cap V_k.$$

Clearly, V_k is compact and convex, C_k is continuous and gr Q_k is open with $y_j \notin \mathrm{co}\, Q_k(\lambda, x, y, \nu)$ for $j = k - I$.

“Shareholders” For agent $k \in \{I + J + 1, \ldots, I + 2J\}$, the strategy set, $V_k \subset \mathbb{R}^{S+1}$, is defined by

$$V_k = \Delta^S_+,$$

the constraint correspondence, $C_k : V \to V_k$, is defined by

$$C_k(\lambda, x, y, \nu) = \{\nu'_j \in V_k \mid \nu'_j \in \langle Y - Q \rangle^{\perp}\}$$

where $j = k - I - J$. Let the correspondence $F_i : V_i \times V_k \to V_k$ where $i \in \{1, \ldots, I\}$ be defined by

$$F_i(x_i, \nu_j) = \{\nu'_j \in V_k \mid \|\nu'_j - \mu_i(x_i)\| < \|\nu_j - \mu_i(x_i)\|\}$$

where $j = k - I - J$ and let the correspondence $G_j : \prod_{i=1}^{I} V_i \times V_k \times V_k \to \{1, \ldots, I\}$ be defined by

$$G_j(x, \nu_j, \nu'_j) = \{i \in \{1, \ldots, I\} \mid \nu'_j \in F_i(x_i, \nu_j)\}.$$

Then the preference correspondence, $Q_k : V \to V_k$, is defined by

$$Q_k(\lambda, x, y, \nu) = \begin{cases} \emptyset, & \text{for } \sum_i \eta_{ij}(\lambda, x, y)^+ = 0 \\ \left\{\nu'_k \in V_k \,\middle|\, \dfrac{\sum_{i \in G_j(\nu_k, \mu, \nu'_k)} \eta_{ij}(\lambda, x, y)^+}{\sum_i \eta_{ij}(\lambda, x, y)^+} > \rho \right\} & \text{for } \sum_i \eta_{ij}(\lambda, x, y)^+ > 0 \end{cases}$$

where $j = k - I - J$ and $\eta_i : V_0 \times \prod_{i=1}^{I} V_i \times \prod_{j=1}^{J} Z_j \to \mathbb{R}^J$ is defined by

$$\eta_i(\lambda, x, y) = \arg\min \|x_i - \omega_i - Q\delta_i - (Y - Q)\eta_i\|, \ \text{s.t. } \eta_i \in \mathbb{R}^J.$$

Clearly, V_k is compact and convex and C_k is continuous. Lemma 1 below shows that gr Q_k is open and that $\nu_j \notin \mathrm{co}\,(Q_k(\lambda, x, y, \nu) \cap C_k(\lambda, x, y, \nu))$ for $j = k - I - J$.

Lemma 1 The preference correspondence for shareholders, $Q_k : V \to V_k$ where $k \in \{I + J + 1, \dots, I + 2J\}$, has the following properties

- $\operatorname{gr} Q_k$ is open, and,
- $\pi_j \notin \operatorname{co}(Q_k(\lambda, x, y, \pi) \cap C_k(\lambda, x, y, \pi))$ for $k = I + J + j$.

Proof: "$\operatorname{gr} Q_k$ is open" Suppose that $(x_i(n))_{t \in N} \in V_i$ converges to $x_i \in V_i$ and that $(\mu_i(x_i(n)))_{t \in N} \in \Delta^S_+$ converges to $\mu_i \neq \mu_i(x_i) \in \Delta^S_+$. Then there exists $x'_i \in P_i(x_i)$ such that $\mu_i \cdot (x'_i - x_i) \leq 0$ so there exists $x''_i \in P_i(x_i)$ such that $\mu_i \cdot (x''_i - x_i) < 0$. Therefore there exists $N \in \mathbb{N}$ such that if $n \geq N$ then $\pi_i(x_i(n)) \cdot (x''_i - x_i(n)) < 0$ and $x''_i \in P_i(x_i(n))$ according to (a.3). This is a contradiction thus $\mu_i : V_i \to \Delta^S_+$ is continuous. Clearly, if $\mu_i : V_i \to \Delta^S_+$ is continuous for all i then $\operatorname{gr} Q_k$ is open due to the definition of $Q_k : V \to V_k$.

"$\pi_j \notin \operatorname{co}(Q_k(\lambda, x, y, \pi) \cap C_k(\lambda, x, y, \pi))$ for $k = I + J + j$" Let $[r] \in \mathbb{Z}$ be defined by $[r] \leq r < [r] + 1$ for all $r \in \mathbb{R}$ and let $L_m = \sum_i [\eta_{ij}(\lambda, x, y)^+ m] \leq m \sum_i \eta_{ij}(\lambda, x, y)^+$ artificial consumers be defined by

$$T_l = U_i(\pi_j, x_i)$$

for all $l \in \{\sum_{i' < i} [\eta_{i'j}(\lambda, x, y)m] + 1, \dots, \sum_{i' \leq i} [\eta_{i'j}(\lambda, x, y)m]\}$ and $m \in \mathbb{N}$ provided that $m\eta_{ij}(\lambda, x, y) \geq 1$. Let $t_j : \Delta^S_+ \to \{1, \dots, L_m\}$ be defined by

$$t_j(\pi'_j) = \{l \in \{1, \dots, L_m\} | \eta_{ij}(\lambda, x, y)^+ > 0 \text{ and } \pi'_j \in T_l\}$$

and let $R_j \subset \Delta^S_+$ be defined by

$$R_j = \{\pi'_j \in \Delta^S_+ | |t_j(\pi'_j)| > \rho \sum_i \eta_{ij}(\lambda, x, y)^+ m\}.$$

If $\pi'_j \in Q_k(\lambda, x, y, \pi)$ then there exists $\varepsilon > 0$ such that

$$\varepsilon = \frac{\sum_{i \in G_j(\pi_j, x, \pi'_j)} \eta_{ij}(\lambda, x, y)^+}{\sum_i \eta_{ij}(\lambda, x, y)^+} - \rho$$

due to the definition of $Q_k : V \to V_k$. Therefore, $\pi'_j \in R_j$ provided that $m > I/(\varepsilon \sum_i \eta(\lambda, x, y)^+)$ so $Q_k(\lambda, x, y, \pi) \subset \cup_m R_j$.

Clearly, $\pi_j \notin \mathrm{co}\,(F_i(\pi_j, x_i) \cap C_k(\lambda, x, y, \pi))$ due to the construction of $F_i : \Delta_+^S \times V_i \to \Delta_+^S$ therefore $\pi_j \notin \mathrm{co}\,(R_j \cap C_k(\lambda, x, y, \pi))$ for all m because $\dim C_k(\lambda, x, y, \pi) = S - J$ and $\rho \geq (S - J)/(S - J + 1)$ hence $\pi_j \notin \mathrm{co}\,(Q_k(\lambda, x, y, \pi) \cap C_k(\lambda, x, y, \pi))$ according to Greenberg (1979).

Q.E.D.

Step 2: construction of correspondence and existence of fixed point

Let $K = \{0, \dots, I + 2J\}$ then V_k is compact and convex for all $k \in K$ and $V = \prod_{k \in K} V_k$. Let the map $f_k : V \times V_k \to \mathbb{R}_+$ be defined by

$$f_k(z, z_k') = \min_{(v, v_k') \in (\mathrm{gr}\, Q_k)^c} \|(z, z_k') - (v, v_k')\|,$$

the correspondence $g_k : V \to V_k$ by

$$g_k(z) = \arg\max_{z_k' \in C_k(z)} f_k(z, z_k').$$

Then the correspondence, $h : V \to V$ defined by $h_k(z) = \mathrm{co}\, g_k(z)$ is upper hemi-continuous and compact and convex valued. Therefore there exists $(\lambda^*, x_1^*, \dots, x_I^*, y_1^*, \dots, y_J^*, \pi^*) = z^* \in Z$ such that $z^* \in h(z^*)$ according to the Kakutani fixed point theorem. Hence, $z_k^* \in C_k(z^*)$ and $Q_k(z^*) \cap C_k(z^*) = \emptyset$ because $z_k \notin \mathrm{co}\, Q_k(z) \cap C_k(z)$.

Step 3: existence of ρ-majority stable equilibrium

For consumers, there exists $x_i(\{\omega_i\} + \sum_j \delta_{ij} \mathrm{co\,cl}\, Z_j) \in \mathbb{R}^{S+1}$ such that if $x_i \in B_i(Q, Y)$ and $P_i(x_i) \cap B_i(Q, Y) = \emptyset$ then $x_i \geq x_i(\{\omega_i\} + \sum_j \delta_{ij} \mathrm{co\,cl}\, Z_j)$ according to (a.6) because $\{\omega_i\} + \sum_j \delta_{ij} \mathrm{co\,cl}\, Z_j$ is compact according to (a.7). Therefore there exist $z^* \in h(z^*)$ and $N_C \in N$ such that if $n \geq N_C$ - recall that $V_k = X_i \cap (\{\omega_i\} + \sum_j \delta_{ij} \mathrm{co\,cl}\, Z_j + [-n, n]^{S+1})$ for $k = i$ - then $x_i^* \in B_i(Q^*, Y^*)$ and $\mathrm{co}\, P_i(x_i^*) \cap B_i(Q^*, Y^*) = \emptyset$ thus $x_i^* = \omega_i + Q^* \delta_i + (Y^* - Q^*)\theta_i^*$ for some $\theta_i^* \in [0, 1]^J$.

For consumers, if ¸(n) ! ¸ where ¸^{s}(n) ¸ 1=n and ¸^{s} = 0 for s 2 S^0 ½ S and x_1(n) 2 C_1(¸(n); x(n); y(n); º(n)) and Q_1(¸(n); x(n); y(n); º(n)) \ C_1(¸(n); x(n); y(n); º(n)) = ; then $\mathbf{P}_{s2S^0}$ x_1^s(n) ! 1 according to (a.3) and (a.4) while consumption is bounded from below for all consumers according to (a.6). Therefore, for the auctioneer, there exists N_A 2 N such that if n ¸ N_A - recall that V_k = ¢$^S_+$ \ [1=n; 1]$^{S+1}$ for k = 0 - and z¤ 2 h(z¤) then $\mathbf{P}_i$ $x_i^¤$ = $\mathbf{P}_i$!$_i$ + $\mathbf{P}_j$ $y_j^¤$.

For the ¯rms, if z¤ 2 h(z¤) then $y_j^¤$ 2 arg max º$_j^¤$$y_j$; s.t. y_j 2 co cl Z_j therefore $y_j^¤$ 2 arg max º$_j^¤$$y_j$; s.t. y_j 2 Y_j and Y_j ½ f$y_j^¤$g + hº$_j^¤$i ¡ R_+^{S+1}.

Lemma 2 If z¤ 2 h(z¤) and n ¸ N_C then $P_j^½$(x¤; µ$_j^¤$; $y_j^¤$) = ;.

Proof: Suppose that n ¸ N_C, if $x_i^¤$ + (y_j ¡ $y_j^¤$)µ$_{ij}^¤$ 2 P_i($x_i^¤$) and µ$_{ij}^¤$ > 0 then ¹$_i$($x_i^¤$) ¢ (y_j ¡ $y_j^¤$) > 0 and if ¹$_i$($x_i^¤$) ¢ (y_j ¡ $y_j^¤$) 6 0 and µ$_{ij}$ > 0 then $x_i^¤$ + (y_j ¡ $y_j^¤$) 62 P_i($x_i^¤$). Hence, if

$$\frac{\mathbf{P}_{i2H_j(v_j)}\, µ_{ij}^{¤+}}{\mathbf{P}_i\, µ_{ij}^{¤+}} \;.\; ½$$

for all v_j 2 hº$_j^¤$i$^?$ where

$$H_j(v_j) \;=\; \text{f}i \;2\; \text{f}1; : : : ; I\text{g}j\,¹_i(x_i^¤) ¢ v_j > 0\text{g}$$

then $P_j^½$(x¤; µ$_j^¤$; $y_j^¤$) = ; because Y_j ½ f$y_j^¤$g + hº$_j^¤$i$^?$ ¡ R_+^{S+1}. Thus, hv_ji$^?$ separates $H_j(v_j)$ from the rest of the i's in the sense that $H_j(v_j)$ is above hv_ji$^?$ while the rest of the i's are below or on hv_ji$^?$, i.e. i 2 $H_j(v_j)$ if and only if ¹$_i$($x_i^¤$) ¢ v_j > 0.

For v_j 2 hº$_j^¤$i$^?$ suppose that $\mathbf{P}_s$ v_j^s < 1 without loss of generality and let (p(n))$_{n2N}$ 2 ¢ where ¢ = f¸ 2 R^{S+1}j $\mathbf{P}_s$ ¸^{s} = 1g be de¯ned by

$$p(n) \;=\; \frac{1}{n + \mathbf{P}_s\, v_j^s}(nº_j^¤ + v_j):$$

for all n then (p(n))$_{n2N}$ converges to º$_j^¤$. Let (q(n))$_{n2N}$ 2 hp(n) + º$_j^¤$i$^?$ be de¯ned by

$$q(n) \;=\; (p(n) \;¡\; º_j^¤) \;¡\; \frac{(p(n) \;¡\; º_j^¤) ¢ (p(n) + º_j^¤)}{(p(n) + º_j^¤) ¢ (p(n) + º_j^¤)}(p(n) + º_j^¤):$$

for all n. Then some tedious calculations show that $(nq(n))_{n\in\mathbb{N}}$ converges to v_j and $\langle q(n)\rangle^\perp$ separates $G_j(\gamma^*_j; x^*; p(n))$ from the rest of the i's in the sense that $G_j(\gamma^*_j; x^*; p(n))$ is above $\langle q(n)\rangle^\perp$ while the rest of the i's are below or on $\langle q(n)\rangle^\perp$. Moreover there exists $N \in \mathbb{N}$ such that if $n \geq N$ then $\langle v_j\rangle^\perp$ separates $G_j(\gamma^*_j; x^*; p(n))$ from the rest of the i's in the sense that $G_j(\gamma^*_j; x^*; p(n))$ is above $\langle v_j\rangle^\perp$ while the rest of the i's are below or on $\langle v_j\rangle^\perp$. Thus if $n \geq N$ then $G_j(\gamma^*_j; x^*; p(n)) = H_j(v_j)$. Therefore, $P^\rho_j(x^*; \theta^*_j; y^*_j) = \emptyset$ because $Q_{I+J+j}(z^*) = \emptyset$.

Q.E.D.

For shareholders, if $z^* \in h_{I+J+j}(z^*)$ then $P^\rho_j(x^*; \theta^*_j; y^*_j) = \emptyset$ provided that $n \geq N_C$ according to lemma 2. Thus, if $\rho \geq (S - J)/(S - J + 1)$ and $n \geq \max\{N_A; N_C\}$ then a ρ-majority stable equilibrium exists.

Part 2: $\rho \geq B/(B + 1)$

The variables to be determined are state prices, $\lambda \in \Delta^S_+$, consumption bundles for consumers, $x = (x_i)^I_{i=1} \subset \prod^I_{i=1} X_i$, and production plans for firms, $y = (y_j)^J_{j=1} \subset \prod^J_{j=1} Y_j$.

Let strategy sets, constraint correspondences and preference correspondences be defined as in part 1 of the proof for $k \in \{0; 1; \ldots; I\}$.

“Firms” For agent $k \in \{I + 1; \ldots; I + J\}$, the strategy set, $V_k \subset \mathbb{R}^{S+1}$, is defined by

$$V_k = \text{co cl } Z_j$$

where $j = k - I$, the constraint correspondence, $C_k : V \to V_k$, is defined by

$$C_k(\lambda; x; y;) = V_k$$

and the preference correspondence, $Q_k : V \to V_k$, is defined by

$$Q_k(\lambda; x; y) = P^\rho_j(x; \eta_j(\lambda; x; y); y_j) \cap V_k.$$

Clearly, V_k is compact and convex, C_k is continuous and gr Q_k is open with $y_j \notin \text{co}\, Q_k(\lambda, x, y)$ for $j = k - l$ according to the proof of theorem 2 in Greenberg (1979).

The rest of the proof follows from part 1. Thus, if $\rho \geq B/(B+1)$ then a ρ-majority stable equilibrium exists.

Part 3: an example showing that the bound is binding

Consider an economy with S consumers with utility functions linear in period zero consumption and log-linear in period 1 consumption. Consumer i is indexed by weights, $\pi_i = (\pi_i^s)_{s=1}^S$ with $\sum_{s=1}^S \pi_i^s = 1$, on consumption in different states. The utility function of consumer i is:

$$u_i(x_i) = x_i^0 + \sum_{s=1}^{S} \pi_i^s \log x_i^s \quad \text{with} \quad \begin{cases} \pi_i^s = \varepsilon & \text{if} \quad s \neq i \\ \pi_i^i = 1 - (S-1)\varepsilon \end{cases} \tag{1}$$

where $\varepsilon \in]0, 1 - 1/(S-1)[$ is small. Although these utility functions do not satisfy assumption (a.6), since the argument is local they can be easily extended outside the relevant domain to fulfill this assumption. All consumers are endowed with identical initial shares of the J firms: $\delta_{ij} = 1/S$, for all i, j, and the same vector of initial resources: $\omega_i = (\omega, 0, \dots, 0)$, all i.

All J firms have their sets of efficient production plans included in the same $(S-1)$-dimensional linear subspace:

$$Y = \left\{ y = (y^0, y^1, y^2, \dots, y^S) \in R^S \mid \sum_{s=0}^{S} y^s = 0 \text{ and } y^0 = -1 \right\}.$$

Define the production plans $y = (y_j)_{j=1}^J$ by:

$$y_j^s = \begin{cases} -1 & \text{for } s = 0 \\ 1 & \text{for } s = j \\ 0 & \text{otherwise} \end{cases}$$

for $j \le J-1$ and for firm J

$$y^s_J = \begin{cases} -1 & \text{for } s = 0 \\ 1/(S-J+1) & \text{for } s \in \{J;\dots;S\} \\ 0 & \text{otherwise} \end{cases}$$

Next, define $\tilde{y} = (\tilde{y}_j)_{j=1}^J$ such that $\tilde{y}_J = y_J$ and $\tilde{y}_j = \alpha y_j + (1-\alpha)y_J$ for $j \le J-1$, with $\alpha = J/S$. Let, for all j, $Z_j = Y \cap B(\tilde{y}_j;\delta)$ where $B(\tilde{y}_j;\delta)$ stands for the ball with center $\tilde{y}_j$ and radius δ. This way, an $(\varepsilon;\delta)$-economy is defined.

Observation 1 For all η, there exists $(\varepsilon;\delta)$ such that the $(\varepsilon;\delta)$-economy does not have a ρ-majority stable equilibrium for $\rho < (S-J)/(S-J+1) - \eta$.

Consider the $(\varepsilon;0)$-economy. It is now shown that there is a unique ρ-majority stable equilibrium (for all ρ since $\delta = 0$ implies there is no alternative production plan), $(q^*;\theta^*;y^*)$, with $y^* = \tilde{y}$ and $q^* = \beta 1_J$ where $\beta = S/J - 1$.

For the announced production plans y^*, the expression of the utility level of agent i buying, at price q^*, the portfolio $(\theta_{ij})_{j=1}^J$ is:

$$\underbrace{\omega + \beta\frac{J}{S} - (\beta+1)\sum_{j=1}^{J}\theta_{ij}}_{x_i^0} + \sum_{s=1}^{J-1}\pi_i^s \log \underbrace{[\alpha\theta_{is}]}_{x_i^s; s\le J-1} + \pi_i^V \log \underbrace{\left[\frac{\theta_{iJ} + (1-\alpha)\theta_{iU}}{S-J+1}\right]}_{x_i^s; s \ge J};$$

where $U = \{1;\dots;J-1\}$, $V = \{J;\dots;S\}$, $\pi_i^V = \sum_{s\in V}\pi_i^s$ and $\theta_{iU} = \sum_{j\in U}\theta_{ij}$.

First-order conditions of this maximization problem (optimal portfolio choice) gives:

$$\forall s \le J-1: \ \frac{\pi_i^s}{\theta_{is}} + \frac{(1-\alpha)\pi_i^V}{\theta_{iJ} + (1-\alpha)\theta_{iU}} = \beta + 1; \ \text{and} \ \frac{\pi_i^V}{\theta_{iJ} + (1-\alpha)\theta_{iU}} = \beta + 1;$$

which in turn yields:

$$\forall s \le J-1: \ \theta_{is} = \frac{\pi_i^s}{\alpha(\beta+1)}; \ \text{and} \ \theta_{iJ} = \frac{\pi_i^V}{\beta+1} - \frac{1-\alpha}{\alpha}\frac{\pi_i^U}{\beta+1}:$$

It is easily checked that stock markets clear, as well as markets for good, for the chosen values of $\alpha = J/S$ and under the equilibrium price $\beta = S/J - 1$. Then the equilibrium portfolio is:

$$\forall s \leq J - 1: \ \theta^*_{is} = \pi_i^s; \text{ and } \theta^*_{iJ} = \frac{J}{S}\pi_i^V - (1 - \frac{J}{S})\pi_i^U;$$

which is such that $\sum_{j=1}^{J} \theta^*_{ij} = \frac{J}{S}$ for all i.

Suppose now that firm J is given the opportunity to propose a small change of its production plan. For ε small enough, one has $\theta^*_{iJ} > 0$ for $J \leq i \leq S$ and $\theta^*_{iJ} < 0$ for $0 \leq i \leq J - 1$. Hence, only the $S - J + 1$ last consumers have a positive quantity of shares in firm J and consequently they are the only ones to vote, with the same voting weights. The utility function of consumer i, $J \leq i \leq S$, has been constructed such that, at this symmetric equilibrium, consumer i supports a (technically possible) change from y_J to y'_J in Z_J if and only if $y^i_J \leq y'^i_J$, i.e. any change that yields more in state i. For example, $y'_J = y_J + (0, \dots, 0, -\varepsilon, \varepsilon/(S - J), \dots, \varepsilon/(S - J))$ gets the support of the last $S - J$ shareholders/shares. Hence, (q^*, θ^*, y^*) is not stable for any super majority rule of size smaller than $(S - J)/(S - J + 1)$. Subject to the obvious upper hemi-continuity of the equilibrium correspondence in the present setup, any ρ-majority stable equilibrium of the (ε, γ)-economy, for ε and γ small enough, is such that $\rho > (S - J)/(S - J + 1) - \eta$. Finally, note that $S - J \leq B = S - 1$ so there is no need to consider the other case which is more obvious.

Proof of Corollary 1

The variables to be determined are prices, $p \in \Delta^J_+$, consumption bundles for consumers, $x = (x_i)_{i=1}^I \subset \prod_{i=1}^I X_i$, and production plans for firms, $y = (y_j)_{j=1}^J \subset \prod_{j=1}^J Y_j$.

"Auctioneer" For agent $k = 0$, the strategy set, $V_k \subset \mathbb{R}^{J+1}$, is defined by

$$V_k = \Delta^J_+ \cap [\frac{1}{n}; 1]^{J+1}$$

where $n \in N$, the constraint correspondence, $C_k : V \to V_k$, is defined by

$$C_k(p; x; y) = V_k$$

and the preference correspondence, $Q_k : V \to V_k$, is defined by

$$Q_k(p; x; y) = \{p' \in V_k | (p'_0 - p_0)(\sum_i x'_i - \sum_i \omega'_i - \sum_j y'_j) + \sum_j (p'_j - p_j)(\sum_i \eta_{ij}(p; x; y) - 1) > 0\}.$$

where $q_j = p_j/p_0$ for all j and $\eta_i : V_0 \times \prod_{i=1}^{I} V_i \times \prod_{j=1}^{J} Z_j \to [0; 1]^J$ is defined by

$$\eta_i(p; x; y) = \arg\min \| x_i - \omega_i - Q\delta_i - (Y - Q)\eta_i \|, \text{ s.t. } \eta_i \in [0; 1]^J$$

for all i. Clearly, V_k is compact and convex, C_k is continuous and gr Q_k is open with $p \notin \text{co } Q_k(p; x; y)$.

"Consumers" As in part 1 in the proof of theorem 1 - restricting portfolios to $[0; 1]^J$ and disregarding γ.

"Firms" As in part 2 in the proof of theorem 1 - restricting portfolios to $[0; 1]^J$ in the definition of $P_j^{\rho}(x; \theta_j; y_j)$, replacing θ_j with $\eta_j(p; x; y)$ and disregarding γ.

The rest of the proof follows from the last part of part 1 in the proof theorem 1.

Proof of Corollary 3

Proof: Follows from the proof of Theorem 1 with minor changes provided that $x_i \in \breve{B}_i(Q; Y)$ and $P_i(x_i) \cap \breve{B}_i(Q; Y)$ imply that $y_j \notin \text{co } \breve{U}_{ij}(q; x_i; y_j)$.

Hence, suppose that $(y_j(n))_{n=1}^{N} \in \breve{U}_{ij}(q; x_i; y)$ where $N \in N$ then there exists $(\theta_i(n))_{n=1}^{N}$ such that $x_i + (Y|y_j(n) - Q)\theta_i(n) \in P_i(x_i)$ for all $n \in \{1; \ldots; n\}$. Suppose that $y'_j = \sum_n \alpha(n) y_j(n)$ where $\alpha(n) \geq 0$ for all n and

$\sum_n \alpha(n) = 1$ and let θ_j' and $(\beta(n))_{n=1}^N$ where $\beta(n) \geq 0$ for all n and $\sum_n \beta(n) = 1$ be defined by $\alpha(n)\theta_{ij}' = \beta(n)\theta_{ij}(n)$ for all n and $\theta_{ij'}' = \sum_n \beta(n)\theta_{ij'}(n)$ for all $j' \neq j$. Then

$$x_i + (Y|y_j' - Q)\theta_i' = \sum_{n=1}^{N} \beta(n)(x_i + (Y|y_j(n) - Q)\theta_i(n)).$$

Therefore, if $x_i \in \tilde{B}_i(Q, Y)$ and $P_i(x_i) \cap \tilde{B}_i(Q, Y)$ then $y_j \notin \mathrm{co}\,\tilde{U}_{ij}(q, x_i, y_j)$.

It is necessary to bound portfolios to be non-negative in order to ensure that there exists θ_i' and $(\beta(n))_n$ such that $(Y|y_j' - Q)\theta_i' = \sum_n \beta(n)(Y|y_j(n) - Q)\theta_i(n)$.